

TABLE IV. Summary of full electrode disk data.<sup>a</sup>

Shot	$\mu_p$	$\sigma$	$k$	$i_t/i_s$	Tilt factor	Configuration
H-34	0.1233	18.69	2.18	1.06	0.52	1¼ × 0.050 in. $d/l=25$
H-33	0.1284	19.46	2.15	1.08	0.58	
H-79B	0.1242	18.83	2.21	1.05	0.50	½ × 0.025 in. $d/l=20$
80B	0.1413	21.42	2.18	1.05	0.37	
81B	0.2109	32.4	2.30	1.12	0.41	
H-79A	0.1242	18.83	2.11	1.14	0.14	1¼ × 0.075 in. $d/l=16.7$
80A	0.1413	21.42	2.16	1.15	0.16	
99	0.1602	24.28	2.10	1.18	0.18	
81A	0.2109	32.4	2.26	1.17	0.27	
H-104B	0.07043	10.68	1.98	1.16	0.11	
116B	0.1008	15.28	2.05	(b)	0.13	
107B	0.1476	22.37	2.08	1.26	0.04	
109	0.2193	33.8	2.26	1.19	0.35	
H-97B	0.09552	14.48	1.99	1.35	0.05	1¼ × 0.25 in. $d/l=5$
98B	0.1563	23.69	2.07	1.46	0.04	
100B	0.1893	28.9	2.04	1.46	0.07	
101B	0.2393	37.0	1.89	1.78	0.11	
102B	0.3129	49.0	2.11	1.79	0.05	
H-104A	0.07043	10.68	1.95	1.17	0.43	½ × 0.1 in. $d/l=5$
116A	0.1008	15.28	1.96	(b)	0.05	
106	0.1060	16.07	2.02	1.20	0.37	
107A	0.1476	22.37	2.11	1.36	0.11	
109A	0.2193	33.8	2.16	1.43	0.17	
110	0.2886	45.0	2.22	1.31	0.44	
H-97C	0.09552	14.48	2.07	1.33	0.06	½ × 0.125 in. $d/l=4$
98C	0.1563	23.69	2.03	1.47	0.06	
H-49	0.01948	2.95	1.75	(b)	0.27	½ × 0.25 in. $d/l=2$
55	0.01956	2.96	1.70	1.83	0.08	
39	0.02569	3.89	1.70	1.71	0.09	
38	0.03474	5.27	1.73	1.73	0.27	
56A	0.06650	10.08	1.94	1.50	0.18	
B	0.06650	10.08	1.87	1.61	0.06	
C	0.06650	10.08	1.95	1.45	0.10	
97A	0.09552	14.48	1.85	1.87	0.04	
98A	0.1563	23.69	1.75	2.13	0.03	

<sup>a</sup> Definitions of terms and units are the same as those used in Table I. <sup>b</sup>  $i_t$  not recorded due to partial experimental failure.

to cause further distortions to the current waveform. The data from the investigation of the fully electroded disk are shown in Table IV. A surprising result is that the current coefficient for the fully electroded disk is essentially constant for the stress range of 9 to about 25 kbar.

Table V summarizes the current coefficients for the various configurations. For a given stress, the  $k$  values are definitely lower for the smaller  $d/l$  ratio gauges. For gauges with  $d/l \geq 20$  the  $k$  values obtained are the same within experimental error as those obtained for the guard ring. For large  $d/l$ -ratio disks we would expect the current waveform to approach the one-dimensional guard-ring current waveform since both field fringing and unloading wave effects become negligible during the first-wave transit time.

The current-time record of a typical fully electroded disk shows a nonlinear current rise in time after the initial current jump. However, disks with  $d/l \geq 10$  show a linear current rise. For the  $d/l=5$  disk, the current rise is linear for about one-half wave-transit time and then becomes nonlinear. For disks with

TABLE V. Current coefficient for various fully electroded disks. Mean values for stress between 9–25 kbar.

Diameter thickness ratio	$k_{\text{mean}}$
2	1.90
4	2.05
5	2.05
5	2.05
10	2.04
16.7	2.12
20	2.20
25	2.17

$d/l \leq 4$ , the current is nonlinear in time for essentially the entire wave-transit time.

The experiments on  $d/l=2$  disks confirm that there is a stress region below 6 kbar for which the current coefficient does not change. Further, the difference observed in  $k$  between the  $d/l=5$  and  $d/l=16.7$  disks agrees with the determination of Jones *et al.*<sup>1</sup>

For an input stress greater than 25 kbar, considerable distortion to the current-time waveform occurs due to conduction. The distortion is greater for the more bounded disks, indicating that the conduction is more pronounced in the regions of the gauge affected by the unloading waves. For this reason, at stress amplitudes greater than 25 kbar, the bounded disk is an accurate gauge only for the current jump.

Since the current jump is directly affected by the capacitive field fringing of the disk, the values of  $k$  quoted here for the small  $d/l$  disks are strictly applicable only for the electrode configuration used here, which is a ground plane of essentially infinite extent and a positive electrode the same size as the disk. A potting compound of radically different dielectric permittivity could conceivably alter the field fringing behavior.

CONCLUSION

Despite the fact that local strain and resulting polarization are coupled to each other with the speed of light in a piezoelectric medium, time-dependent effects are possible through variable wave velocity, strain relaxation, dielectric permittivity relaxation, or conduction. Our analysis of the data indicates that for stress less than about 25 kbar and for disks in the  $+X$  orientation it is possible to describe all aspects of our data to time-independent physical properties. The most rapid stress application possible with existing gun and explosive-loading techniques within the useful stress range of the gauge is  $10^{-8}$  sec over a ½-in.-diam inner electrode. However, the rate of stress application to a given point on the surface is certainly higher. For the range of stress rates achieved in shock experiments and for stress values below about 25 kbar we conclude that there are no detectable rate effects. Our results define the appropriate physical properties of X-cut

quartz with sufficient accuracy so that the current from quartz disks may be used for detecting stress-time profiles with a stress-amplitude accuracy comparable to other shock detection techniques and with a superior time resolution.

#### ACKNOWLEDGMENTS

The authors are indebted to numerous colleagues at Sandia Laboratory for helpful and stimulating discussions; to W. D. Ingram for specimen preparation and mechanical assistance; and, particularly, to G. E. Ingram who designed the electronic circuitry and developed methods to obtain the precise alignments required.

#### APPENDIX

An analysis is given for the effects of finite strain, dielectric permittivity change, parallel-plate capacitive field fringing and piezoelectric coupling on the current generated in an X-cut quartz disk subjected to a step function of stress.

#### General Expressions

For the particular case of a step function of stress assumptions (a)-(j) we may derive equations for electric field and current. At any time during the first wave transit the disk is divided into two regions: the unstressed region (denoted by subscript 1 hereafter) and the stressed region (denoted by subscript 2 hereafter).

For zero conductivity,

$$D_1 = D_2, \quad (A1)$$

where  $D$  is the electric displacement.

Since an electric short circuit exists between the two electrodes,

$$E_2 l_2 = -E_1 l_1. \quad (A2)$$

We know that

$$l_2 = U_s t, \quad \text{and} \quad l_1 = l - U_s t. \quad (A3)$$

Solving for  $E_2$  in terms of  $E_1$  from Eq. (A2) and substituting this into Eq. (A1), we find that

$$E_1 = P U_s t / \epsilon l, \quad (A4)$$

and

$$E_2 = P(1 - U_s t / l) / \epsilon. \quad (A5)$$

The magnitude of the field in the stressed region of the disk at a stress of 20 kbar and at  $t=0+$  is  $1.13 \times 10^5$  V-cm<sup>-1</sup>. The presence of these large fields requires that the lateral surface of the gauge be exceptionally clean and carefully potted.

From Eqs. (A1) and (1) we find that the current is

$$i = P A U_s / l, \quad 0 < t < l / U_s, \quad (A6)$$

in agreement with Eq. (6).

#### Finite Strain

For the case of finite strain  $s$ ,

$$l_2 = N U_s t, \quad \text{and} \quad l_1 = l - U_s t,$$

where

$$N = 1 - s.$$

Again we obtain the current by solving for  $E_2$  in terms of  $E_1$  and substituting this into Eq. (A1) and (1). The solution is

$$i = P A U_s N [U_s t (N - 1) + l]^{-2}. \quad (A7)$$

Neglecting  $(N - 1)^2$  terms, we find that

$$i_t / i_i = (2N - 1)^{-1}. \quad (A8)$$

From the known elastic stiffness we find the value of this ratio to be 1.048 at 20 kbar. For the strain involved in the usual gauge experiment Eq. (A7) predicts a linear increase in current with the time to a close approximation.

#### Dielectric Permittivity Change

If the dielectric permittivity in the stressed region has a value different from the unstressed region, we examine the effect on the current by solving for the current in the same manner as done previously except to allow  $\epsilon_1 \neq \epsilon_2$ . The solution for current is

$$i = P A U_s l \epsilon_2 / \epsilon_1 [U_s t + \epsilon_2 (l - U_s t)]^2. \quad (A9)$$

The current ratio is

$$i_t / i_i = (\epsilon_2 / \epsilon_1)^2. \quad (A10)$$

For small differences between  $\epsilon_2$  and  $\epsilon_1$  the current increases linearly in time to a close approximation.

#### Time-Dependent Parallel-Plate Capacitive Field Fringing

To analyze the effect of the field fringing for the fields between the stress wave front and the electrodes it is convenient to express the current in terms of the time varying series capacitance of the stressed and unstressed regions. First consider the one-dimensional field case. Because of the electric short-circuit condition

$$q_1 / C_1 = q_2 / C_2, \quad (A11)$$

where  $q$  is the charge on the capacitor and  $C$ , the time-dependent capacitance for one-dimensional fields between the wavefront and the electrodes.

Further,

$$q_2 = P A - q_1. \quad (A12)$$

We can solve for  $q_1$  from Eq. (A12) and (A11). Differentiating the charge with respect to time gives

$$i = dq/dt = P A [C_2 dC_1/dt - C_1 dC_2/dt] [C_1 + C_2]^{-2}. \quad (A13)$$

If we substitute the expressions for the time-depend-